Search for Tensor Interactions in Kaon Decays at DAΦNE

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Abstract

The semileptonic kaon decays $K_{l2\gamma}$ and K_{l3} are considered. We use the most general forms of the matrix elements for these decays. Additional terms could arise as a result of new tensor interactions between quarks and leptons. Such terms have been detected in a recent experiments. The high precision experiments at DAΦNE may be ideal to confirm these results.

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1 Introduction

The testing of the Standard Model now reaches its concluding stage. The most precise data are the ones from the LEP experiments with an accuracy $\sim 1\%$. To describe these data, it is necessary to take into account the electroweak radiative corrections. The data put strict constraints on the new physics i.e. new interactions and new particles. On the other hand the low energy experiments, like particle decays, also can be used as a good test for the V-A interactions.

The recent experiments on semileptonic meson decays [1, 2] point to the possible existence of an admixture of tensor interactions [3, 4], which can help to reconcile [5] the results of the previous experiments [6]. Unfortunately, the experimental data is poor and the experimental status of the form factors determination of the three particles semileptonic decays is obscure [7]. In this connection the construction of the ϕ-factory in Frascati [8] and the K-meson decay experiments would help to check both the Standard Model and the chiral theory [9] with a good accuracy. The comparatively large energy release in the kaon decays would allow to analyze the dependence of form factors on the momentum transfer.

2 $K_{l2\gamma}$ decay

One of the decays, where the existence of tensor interactions in the weak processes was observed, was the radiative semileptonic pion decay $\pi^- \to e^-\tilde{\nu}\gamma$ [1]. The branching ratio $B^{exp} = (1.61 \pm 0.23) \times 10^{-7}$ is smaller than the theoretical ratio $B^{th} = (2.41 \pm 0.07) \times 10^{-7}$ expected in the Standard Model. This fact can be explained by the destructive interference of the standard decay amplitude with the newly introduced tensor amplitude [1, 3]

$$M_T = \frac{eG_F \cos\theta_C}{\sqrt{2}}\ F_T\ \varepsilon^\alpha q^\beta\ \bar{u}(p_e)\sigma_{\alpha\beta}(1-\gamma^5)v(p_\nu). \tag{1}$$

The dimensionless constant F_T describes the strength of the new interaction relative to the ordinary Fermi coupling. Its experimental value is $F_T \approx 5 \times 10^{-3}$.

At the quark level this amplitude can be described by a four fermion interaction of the form

$$\mathcal{L}_T^{\Delta S=0} = \frac{G_F \cos\theta_C}{\sqrt{2}}\ f_T\ \bar{\psi}_u\sigma^{\alpha\beta}\psi_d \cdot \bar{\psi}_e\sigma_{\alpha\beta}(1-\gamma^5)\psi_\nu + \text{h.c.} \tag{2}$$

The relation between F_T and f_T can be obtained in the framework of the relativistic quark model [3]

$$F_T = \frac{1}{3}\,\frac{F_\pi}{m_q}\ f_T \approx 0.13\ f_T,$$

where $F_\pi = 131$ MeV is the pion decay constant, $m_q = 340$ Mev is the constituent quark mass, or by applying the QCD techniques and the PCAC hypothesis [10]

$$F_T = \frac{2}{3}\,\chi\,\frac{<0|\bar{q}q|0>}{F_\pi}\ f_T \approx 0.4\ f_T,$$

here $\chi = -5.7 \pm 0.6$ GeV^{-2} is the quark condensate magnetic susceptibility, $< 0|\bar{q}q|0 >= -(0.24\text{GeV})^3$ is the vacuum expectation value for the quark condensate.

However, the constraints following from the π_{l2} decay [11], forbid the existence of tensor interactions (2) between quarks and leptons with a coupling constant for the tensor interaction $f_T \sim 10^{-2}$, which is necessary to explain the experiment [1].

In ref. [12] an extension of the interaction (2) was proposed including also the strange quark

$$\mathcal{L}_T^{\Delta S=1} = \frac{G_F \sin\theta_C}{\sqrt{2}}\ f_T^1\ \bar{\psi}_u \sigma^{\alpha\beta}\psi_s \cdot \bar{\psi}_l \sigma_{\alpha\beta}(1-\gamma^5)\psi_\nu + \text{h.c.} \tag{3}$$

This interaction can be detected in the kaon decay $K^- \rightarrow l^- \tilde{\nu}\gamma$. However, it is unreliable to make any physical predictions [12] on the base of the contradictory situation of the π_{l2} decay.

This problem was solved in ref. [4], where a new tensor interaction with the coupling constant $f_t \sim 0.1$ was introduced

$$\mathcal{L}_T = -\frac{G_F}{\sqrt{2}}\ f_t\ \bar{\psi}_u \sigma^{\alpha\lambda}\psi_{d(\theta)}\ \frac{Q_\alpha Q^\beta}{Q^2}\ \bar{\psi}_l \sigma_{\beta\lambda}(1-\gamma^5)\psi_\nu + \text{h.c.}\ , \tag{4}$$

dependent on the momentum transfer to the lepton pair Q, where $\psi_{d(\theta)} = \cos\theta_C\ \psi_d + \sin\theta_C\ \psi_s$. This interaction appears effectively through the exchange of massive tensor particles. In ref. [4] a mechanism for providing mass of the tensor particles was proposed, which leads to a pole Q^2 in the interaction (4). For particle decays Q^2 is always positive and does not lead to difficulties. In scattering processes there exists a kinematic region with $Q^2 = 0$. This could lead to a diffraction peak, which, however, had not been observed at the neutrino scattering experiments [13]. This difficulty can be avoided if another mass generating mechanism is used for the tensor particles.

The results of ref. [4] remain valid if the interaction (4) changes its form

$$\mathcal{L}_{ql} = -\frac{G_F}{\sqrt{2}}\ f_t\ \bar{\psi}_u \sigma^{\alpha\lambda}\psi_{d(\theta)}\ \frac{Q_\alpha Q^\beta}{m_\pi^2}\ \bar{\psi}_l \sigma_{\beta\lambda}(1-\gamma^5)\psi_\nu + \text{h.c.}\ , \tag{5}$$

by introducing the parameter m_π^2 which is of the order of the momentum transfer in the pion or kaon decay. This interaction between quarks and leptons leads to an additional tensor term M_T in the total matrix element of the radiative semileptonic kaon decay $K^+ \rightarrow l^+ \nu\gamma$

$$M = M_{IB} + M_{SD} + M_T, \tag{6}$$

where

$$M_{IB} = i\frac{eG_F \sin\theta_C}{\sqrt{2}}\ F_K\ m_l\ \varepsilon_\alpha\ \bar{u}(p_\nu)(1+\gamma^5)\left[\frac{p^\alpha}{pq} - \frac{2p_l^\alpha + i\sigma^{\alpha\beta}q_\beta}{2p_l q}\right] v(p_l) \tag{7}$$

is a QED correction (inner bremsstrahlung) to the $K^+ \rightarrow l^+\nu$ decay with the kaon decay constant $F_K = 160$ MeV, and

$$M_{SD} = -\frac{eG_F \sin\theta_C}{\sqrt{2}m_K}\varepsilon^\alpha \left[F_V e_{\alpha\beta\rho\sigma} p^\rho q^\sigma + iF_A(pq \cdot g_{\alpha\beta} - p_\alpha q_\beta)\right] \bar{u}(p_\nu)\gamma^\beta(1-\gamma^5)v(p_l) \tag{8}$$

is a structure dependent amplitude parametrized by two form factors F_V and F_A; $\varepsilon^\alpha(q)$ is the photon polarization vector with $\varepsilon^\alpha q_\alpha = 0$, and

$$M_T = -\frac{eG_F \sin\theta_C}{\sqrt{2}m_K^2}\ F_T \left\{Q^2\ \varepsilon^\alpha q^\beta + [(\varepsilon Q)q^\alpha - (qQ)\varepsilon^\alpha]\ Q^\beta\right\} \bar{u}(p_\nu)(1+\gamma^5)\sigma_{\alpha\beta}v(p_l), \tag{9}$$

where $Q = p - q = p_\nu + p_l$ is the momentum transfer to the lepton pair.

We choose the kinematical variables as the conventional quantities $x = 2pq/m_K^2$ and $y = 2pp_l/m_K^2$. The differential decay width is

$$\frac{\mathrm{d}^2\Gamma}{\mathrm{d}x\mathrm{d}y} = \frac{\alpha}{2\pi}\ \frac{\Gamma_{K\to l\nu}}{(1-r_l)^2}\ \rho(x,y). \tag{10}$$

Here $r_l \equiv (m_l/m_K)^2$ and the Dalitz plot density $\rho(x,y)$ is given by

$$\rho(x,y) = \rho_{IB}(x,y) + \rho_{SD}(x,y) + \rho_{IBSD}(x,y) + \rho_T(x,y) + \rho_{IBT}(x,y) + \rho_{SDT}(x,y), \tag{11}$$

where

$$\begin{aligned}
&\rho_{IB} = IB(x,y), \quad \rho_{SD}(x,y) = a_l^2\ \left[(1+\gamma_A)^2\ SD^+(x,y) + (1-\gamma_A)^2\ SD^-(x,y)\right], \\
&\rho_{IBSD} = 2a_l\sqrt{r_l}\left[(1+\gamma_A)G^+(x,y) + (1-\gamma_A)G^-(x,y)\right],\ \rho_T(x,y) = a_l^2\gamma_T^2\ T(x,y), \\
&\rho_{IBT} = 2a_l\gamma_T\ I(x,y), \quad \rho_{SDT} = 2a_l^2\gamma_T\sqrt{r_l}\left[(1+\gamma_A)J^+(x,y) + (1-\gamma_A)J^-(x,y)\right].
\end{aligned} \tag{12}$$

We introduce the following constants

$$a_l = \frac{m_K^2}{2F_K m_l}\ F_V, \qquad \gamma_A = \frac{F_A}{F_V}, \qquad \gamma_T = \frac{F_T}{F_V},$$

where the form factor F_V can be estimated from the PCAC hypothesis and the axial anomaly as $F_V = m_K/(4\pi^2 F_K)$. The theory does not give exact predictions for the axial F_A and tensor F_T form factors, which should be determined experimentally. The explicit form of the functions $IB(x,y)$, $SD^\pm(x,y)$, $G^\pm(x,y)$, $T(x,y)$, $I(x,y)$ and $J^\pm(x,y)$ are given in the appendix.

In the radiative semileptonic pion decay $\pi_{e2\gamma}$ the constant $a_e^\pi = m_\pi^3/8\pi^2F_\pi^2 m_l \approx 4$ and the densities ρ_{IB} and ρ_{SD} give almost the same contribution to the decay rate. The densities ρ_{IBSD} and ρ_{SDT} are negligibly small from chirality considerations. Therefore the interference term ρ_{IBT} is not suppressed and gives a considerable contribution to the decay rate. Such kind of a contribution with a negative sign (destructive interference) has been observed experimentally [1].

In the kaon decay $K^+ \to e^+\nu_e\gamma$ the constant $a_e^K \approx 120$ is thirty times as large as the corresponding value a_e^π for the pion decay. This leads to a considerable decrease in the relative contribution of ρ_{IB} and of the corresponding interference term ρ_{IBT}. In case of the radiative kaon decay to the muon $K^+ \to \mu^+\nu_\mu\gamma$ the constant is $a_\mu^K \approx 0.5$, and the leading contribution is that of ρ_{IB}. In this decay the interference contribution

ρ_{IBSD} is essential due to the large muon mass, and ρ_{IBT} is suppressed by the small constant a_μ^K.

We can make the following conclusion: The radiative semileptonic pion decay $\pi_{e2\gamma}$ is an ideal process where the tensor interaction reveals with its full strength. In the radiative semileptonic kaon decays $K_{l2\gamma}$ the tensor interaction effect is suppressed. Moreover, in the kaon decays there exists a strong background from the processes K_{l3}. Therefore, in order to avoid the contribution from these processes the experiments are usually made in the narrow kinematic region $y \approx (1 + r_l)$. In this kinematic region the contribution to the $K_{e2\gamma}$ decay ¿from the tensor interactions is negligible, and actually the experiment is sensitive only to the term SD^+. In case of the $K_{\mu 2\gamma}$ decay the contributions from SD^+, J^+ and T are considerable, but they have the same distribution on the variable x at $y = (1 + r_\mu)$ and can not be separated.

There is still another reason for the disability to make more decisive conclusions about the existence of tensor interactions while analyzing the radiative kaon decay. As a result of the large energy release, the resonance exchange with strangeness [14], can give a considerable contribution to the vector and axial form factors but this contribution is only roughly estimated [15]. Therefore, in order to identify the effects due to tensor couplings, one first has to pin down the contribution from higher-order effects in chiral perturbation theory. It is not an easy task to make the estimations with the required accuracy.

3 K_{l3} decay

One of the brightest demonstrations of the existence of the new interactions is the K_{l3} decay: $K^+ \to \pi^o l^+ \nu$. The most general Lorentz invariant form of the matrix element of this decay is [16]

$$M = \frac{G_F \sin\theta_C}{\sqrt{2}}\ \bar{u}(p_\nu)(1+\gamma^5)\left\{ m_K F_S - \frac{1}{2}\left[(P_K + P_\pi)_\alpha f_+ + (P_K - P_\pi)_\alpha f_-\right]\gamma^\alpha \right.$$
$$\left. + i\frac{F_T}{m_K}\sigma_{\alpha\beta}P_K^\alpha P_\pi^\beta \right\} v(p_l). \qquad (13)$$

It consists of a scalar, a vector and a tensor terms, where the form factors F_S, $f_\pm$ and F_T are functions of the squared momentum transfer to leptons $Q^2 = (P_K - P_\pi)^2$. As should be expected ¿from the W-boson exchange, there exists no compelling evidence for terms other than those of a pure vector nature. The contribution of the electroweak radiative corrections to the scalar and tensor form factors is negligibly small. Therefore, the appearance of a considerable nonzero scalar and tensor form factors [16, 2] points to deviations from the standard V-A interaction.

The term in the vector matrix element with $f_-(Q^2)$ can be reduced (using Dirac equation) to the scalar form factor:

$$(P_K - P_\pi)_\alpha\ \bar{u}(p_\nu)(1+\gamma^5)\gamma^\alpha v(p_l) = -m_l\ \bar{u}(p_\nu)(1+\gamma^5)v(p_l).$$

In the same way the tensor matrix element can be reduced to a vector and a scalar matrix elements with a distinctive dependence on the momentum of the latter:

$$\begin{aligned}2iP_K^\alpha P_\pi^\beta\ \bar{u}(p_\nu)(1+\gamma^5)\sigma_{\alpha\beta}v(p_l) &= -m_l\ (P_K+P_\pi)_\alpha\ \bar{u}(p_\nu)(1+\gamma^5)\gamma^\alpha v(p_l)\\ &\quad +(P_K+P_\pi)_\alpha\ (p_\nu-p_l)^\alpha\ \bar{u}(p_\nu)(1+\gamma^5)v(p_l).\end{aligned}$$

This leads effectively to a redefinition of f_+: $V = f_+ + (m_l/m_K)\ F_T$,
and F_S: $S = F_S + (m_l/2m_K)\ f_- + (P_K+P_\pi)_\alpha\ (p_\nu - p_l)^\alpha\ /(2m_K^2)\ F_T$.

Therefore, the Dalitz plot density in the rest frame of the kaon

$$\rho(E_\pi, E_l) = A\cdot|V|^2 + B\cdot \mathrm{Re}(V^*S) + C\cdot|S|^2 \tag{14}$$

is expressed through V and

$$S = F_S + \frac{m_l}{2m_K}\ f_- + \frac{1}{m_K}\ \left[(E_\nu - E_l) + \frac{m_l^2}{2m_K}\right]\ F_T.$$

Here

$$A = m_K\ (2E_lE_\nu - m_K\Delta E_\pi) - m_l^2\ (E_\nu - \frac{1}{4}\ \Delta E_\pi),$$

$$B = m_l\ m_K\ (2E_\nu - \Delta E_\pi), \qquad C = m_K^2\ \Delta E_\pi.$$

where $\Delta E_\pi = E_\pi^{max} - E_\pi$.

One can see that the study of the $K_{\mu3}$ decay mode alone cannot give a limit on F_S. For the K_{e3} decay, ratio $m_e/m_K \sim 10^{-3}$ is small. The interference terms between the vector and the scalar matrix elements are also small. This makes it possible to separate the corresponding contributions from various matrix elements. A characteristic feature of the tensor matrix element contribution is the existence of a minimum, in the case when the energies of neutrino E_ν and the positron E_e are equal.

This is well noticed in the c.m. frame of the lepton pair. The Dalitz plot density, given in (14), can be transformed to the dilepton c.m. system

$$\rho(E_\pi, \cos\theta) \propto \frac{\varepsilon^5 p_\pi}{E_K}\left\{|m_K^2 F_S + \varepsilon p_\pi F_T\cos\theta|^2 + p_\pi^2|f_+|^2\sin^2\theta\right\}, \tag{15}$$

where θ is the angle between π^o and e^+ in the same system, $p_\pi = \sqrt{E_\pi^2 - m_\pi^2}$, $E_K = \sqrt{p_\pi^2 + m_K^2}$ and $\varepsilon = E_K - E_\pi = \sqrt{Q^2}$.

In ref. [17] a method for the separation of the form factors was proposed using integration over the pion spectrum and fitting to the distribution of $\cos\theta$. However, in case of F_S close to zero, the tensor contribution dependence on $\cos^2\theta = 1 - \sin^2\theta$ could be absorbed into the vector contribution, and the separation could not be provided in this way. Such a situation occurs in ref. [17]. Meanwhile, we want to note that our formula (15) for $\rho(E_\pi, \cos\theta)$ differs from the one given in [17]. We want also to point to the lack of consistency in the definition of the form factors f_+ in [17], namely in the eq. (1) and in the formula for $\Gamma(K_{e3})$ at page 241 (in the last formula it is smaller by $\sqrt{2}$).

Fitting Dalitz plot to the distribution $\rho(E_\pi, E_e)$ leads to nonzero values of the form factors $|F_S/f_+(0)| = 0.070 \pm 0.016$ and $|F_T/f_+(0)| = 0.53 \pm 0.10$ [2]. An analysis was provided in terms of linearly parametrized $f_+(Q^2) = f_+(0)\ [1 + \lambda_+\ Q^2/m_\pi^2]$ and constants F_S and F_T. It is important to note here that the separation of the scalar contribution is the most unreliable, as far as any dependence $F_T(Q^2)$ on Q^2 or deviation from linearity in $f_+(Q^2)$ imitates the existence of a nonzero scalar form factor.

Indeed such a dependence arises, if one relies on the new tensor interaction between quarks and leptons (5). In the framework of the relativistic quark model we can get the following relation between F_T and f_t [4]

$$F_T \approx 1.1 \frac{Q^2}{m_\pi^2}\ f_t.$$

We see that the tensor form factor F_T depends on Q^2. Therefore one has to take into account the momentum transfer dependence of the form factors when analyzing experimental data.

4 Conclusion

DAΦNE provides an opportunity to improve our knowledge about the K decays. In this connection we have discussed two modes of the kaon decays $K_{l2\gamma}$ and K_{l3}, where possible deviation from the Standard Model could be seen. By kinematical considerations the tensor interactions can not give a direct contribution to two particles semileptonic meson decay. Therefore we have analyzed meson decays with three particles in the final state. The decays with more particles in the final state lead to a more complicated kinematics and include additional form factors.

We have shown, that for the $K_{l2\gamma}$ decay the contribution ¿from the new tensor interaction is suppressed. Moreover, the strong background from the K_{l3} decay reduces the kinematic region available for analysis. Fortunately, the K_{l3} decay is one, where the presence of the new interactions can be detected. Analyzing the Dalitz plot distribution for the $K^+ \to \pi^o e^+ \nu$ decay we can very well separate the contributions from the vector and tensor terms. The presence of a nonzero tensor or scalar form factor would be a signal of the existence of new interactions in the weak processes.

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Appendix

In this appendix we give an analytical expressions for the functions $IB(x,y)$, $SD^{\pm}(x,y)$, $G^{\pm}(x,y)$, $T(x,y)$, $I(x,y)$ and $J^{\pm}(x,y)$:

$$IB(x,y) = \frac{1-y+r_l}{x^2(x+y-1-r_l)}\left[x^2 + 2(1-x)(1-r_l) - \frac{2xr_l(1-r_l)}{x+y-1-r_l}\right],$$

$$SD^{+}(x,y) = (x+y-1-r_l)\left[(x+y-1)(1-x) - r_l\right],$$

$$SD^{-}(x,y) = (1-y+r_l)\left[(1-x)(1-y) + r_l\right],$$

$$G^{+}(x,y) = \frac{1-y+r_l}{x(x+y-1-r_l)}\left[(x+y-1)(1-x) - r_l\right],$$

$$G^{-}(x,y) = \frac{1-y+r_l}{x(x+y-1-r_l)}\left[(1-x)(1-y) - x + r_l\right],$$

$$T(x,y) = 2(1-x)^2(1-y)(x+y-1) + r_l(1-x)(2-x)(x+2y-2)$$

$$- r_l^2\left[x^2 + 2(1-x)\right],$$

$$I(x,y) = -(1-x)(1-y+r_l),$$

$$J^{+}(x,y) = x\left[(x+y-1)(1-x) - r_l\right],$$

$$J^{-}(x,y) = x(1-x)(1-y+r_l).$$

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